Relationship between the impedance of a multi-pin probe and soil water content

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A b s t r a c t. Based on the theory of the impedance-transform of transmission line, a mathematical model for calculating the impedance of a soil probe with a multi-pin structure was developed. It is found that the impedance of the multi-pin probe may be turned into an inductive-reactance from a capacitive-reactance and vice versa as measurement conditions change. Moreover, the formula for the impedance of a soil probe has a series of periodical discontinuous dots. Three necessary experiments to verify this model were conducted in a laboratory environment. By changing the measurement frequency (50–200 MHz), the length of the pin (5–15 cm) and the number of pins (2–5), some electric properties of the multi-pin probe were confirmed. As an application of the model, a prototype was designed and it achieved a measurement accuracy of volumetric soil water content within $\pm 2\%$ ranging from 0 to 50%.

K e y w o r d s: impedance, dielectric constant, soil water content, multi-pin probe, model

INTRODUCTION

Since Topp *et al.* (1980) presented an empirical equation to describe the relationship between soil moisture and permeability of moist soil, a variety of measurement methods based on dielectric properties of moist soil have become increasingly popular. These measurement technologies rely on the fact that the dielectric constant of water (ca. 80) is significantly greater than that of most soil matrix materials (ca. 3) (Schmugge *et al.*, 1980). The measurement procedure of TDR (Time Domain Reflectrometry) is composed of two steps: 1) measuring propagation velocity of an electromagnetic pulse along a transmission line in soil, and 2) converting this measurement into an estimated value of soil water content through calibration. By using the TDR method, one can get higher accuracy results (ca. $\pm 0.5\%$), but

the main problem for TDR instruments is their high cost. The TDR instrumentation needs fast rise-time with about 100 ps of a step pulse and electromagnetic wave and will travel a distance of 3.3 cm in free space during such a short time. This makes it difficult to design a TDR-instrument and causes the relatively high price of the commercial products (Malicki and Skierucha, 1989). Compared with the TDR method, the high-frequency capacitance sensor to determine soil moisture may give results with the same level of accuracy (Bell et al., 1987; Dean et al., 1987; Dean, 1994; Paltineanu, 1997; Gardner et al., 1998.) Unfortunately, it belongs to a kind of devastating method for soil sampling. The geometric structure of a capacitance sensor is similar to a segment of coaxial line and these sensors are inappropriate for field use (Topp et al., 1980). A relatively new method, which stemmed from the principle of SWR (Standing Wave Ratio), was first reported by Gaskin and Miller (1996), Sun et al. (1999). Like the TDR method, a measured value of SWR needs to be turned into an estimation of soil water content through calibration. One major advantage of the SWR sensor is its lower cost and another is that SWR method belongs to a kind of non-devastating method because the probe has a multi-pin structure. At present, commercially available instruments based on SWR achieved a measurement accuracy within $\pm 2\%$. Although the measurement accuracy of SWR is slightly lower than of TDR, the low cost would allow installation of SWR sensors in large quantities to monitor soil moisture at field scale. In fact, acting as a valuable index to assess soil moisture, the value of SWR depends merely on the change of the impedance of the soil probe but how to calculate the impedance of the soil probe with the multi-pin structure was still a thorny problem

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due to its irregular boundary conditions and its complex field distributions. As far as the measurement frequency is concerned, some investigations were performed (Hipp, 1974; Hoekstra and Delancy, 1974; Hallikainen *et al.*, 1985). The presence of salts in soil water directly influenced the dielectric behavior of soils, especially at frequencies < 30 MHz (Paltineanu, 1997). This study attempted to establish a mathematical model for calculating the impedance of multi-pin probe within a frequency range of 50 to 200 MHz and three necessary experiments to examine this model were conducted. Finally a prototype based on the model was designed.

THEORETICAL APPROACH

Figure 1 shows the field distributions of the two-, three-, four-pin and an ideal coaxial cell given by Zegelin *et al.* (1989). In particular, they pointed out that the field distributions of the three- and four-pin probes provide increasingly better approximations to an ideal coaxial cell in which the equipotentials are concentric circles centred on the inner conductor. This fact means that although three and four-pin



Fig. 1. Dimensionless electric field distributions normal to the direction of probe insertion for a material of uniform dielectric constant.

are classified as an irregular transmission line, their behavior in the field distributions approximate to them of a coaxial line in certain degree. The more pins a probe has, the closer to the behavior of a coaxial line in the field distributions. Thus, an equation concerned with impedance-transformation of any regular transmission line (Mcneil, 1996), such as a parallel line and coaxial line, can be employed:

$$Z_p(l) = Z_c \frac{Z_L + jZ_c \operatorname{tg} \beta l}{Z_c + jZ_L \operatorname{tg} \beta l},$$
(1)

where: Z_p – transform-impedance of a segment of transmission line, Z_L – load-impedance of the transmission line, Z_c – characteristic-impedance of the transmission line, L – length of the transmission line, $j - (-1)^{1/2}$, and β , called the phase coefficient of the transmission line, is defined as:

$$\beta = 2\pi \frac{f}{c} \sqrt{\varepsilon} , \qquad (2)$$

where *c* – velocity of light travelling in free space $(3 \cdot 10^8 \text{ m s}^{-1})$, *f* – the frequency of the oscillator, ε – dielectric constant of the medium filled in the transmission line.

Under the assumption which a multi-pin probe is equivalent to a segment of a transmission line, Z_L should be considered as an open impedance for the probe. Thus,

$$Z_{p}(l)\Big|_{Z_{L}=\infty} = Z_{c} \frac{Z_{l} + jZ_{c} \operatorname{tg} \beta l}{Z_{c} + jZ_{l} \operatorname{tg} \beta l}\Big|_{Z_{L}=\infty} = -jZ_{c} \operatorname{ctg} \frac{2\pi f l \sqrt{\varepsilon}}{c}.$$
(3)

By some elementary transforms, the Eq. (3) can also be expressed as:

$$Z_p(l) = Z_c \operatorname{ctgh}\left(\frac{2\pi f \sqrt{\varepsilon l}}{c} j\right). \tag{4}$$

In order to calculate the impedance Z_p , the characteristic-impedance Z_c of a multi-pin probe has to be solved in advance. It seems too difficult to calculate the Z_c since it is defined as:

$$Z_c = \sqrt{\frac{L_d}{C_d}},$$
 (5)

where L_d – the inductance distributed along a unit length of transmission line, C_d – the capacitance distributed along a unit length of transmission line.

If there are more than two pins, the contour of the field distributions around every pin will become much more complicated than two pins or a coaxial line. Fortunately, an inequality was found to treat this thorny problem. It is known that the characteristic-impedance of parallel line Z'_c can be calculated as:

$$Z'_{c} = \frac{120}{\sqrt{\varepsilon}} \ln \frac{D_{L}}{d_{L}},\tag{6}$$

where $D_L/2$ – span between both pins of parallel line, d_L – diameter of each pin.

As for the characteristic-impedance of coaxial line $Z_c^{"}$, it can be computed by:

$$Z_c'' = \frac{60}{\sqrt{\varepsilon}} \ln \frac{D_c}{d_c},\tag{7}$$

where D_c -diameter of the shield conductor, d_c -diameter of the inner pin.

Choosing $D = D_L = D_c$ and $d_L = d_c$ (Fig. 2), one can ensure:

$$Z'_c \succ Z_c \succ Z''_c. \tag{8}$$

and

$$Z'_c = 2Z''_c. \tag{9}$$

Therefore, the characteristic-impedance of a multi-pin probe can be safely estimated by:

$$Z_c = \xi \frac{60}{\sqrt{\varepsilon}} \ln \frac{D}{d},\tag{10}$$

where the dimensionless coefficient $\boldsymbol{\xi}$ should be fulfilled with

$$1 \ge \xi \ge 2 \tag{11}$$

Replacing Z_c in the Eq. (4) by the Eq. (10), finally one obtained:

$$Z_p(l) = -j\xi \frac{60}{\sqrt{\varepsilon}} \ln \frac{D}{d} \operatorname{ctg} \frac{2\pi f \, l\sqrt{\varepsilon}}{c}, \qquad (12)$$

or

$$Z_p(l) = \xi \frac{60}{\sqrt{\varepsilon}} \ln \frac{D}{d} \operatorname{ctgh} \frac{2\pi f \, l\sqrt{\varepsilon}}{c} \, j. \tag{13}$$



Fig. 2. Geometric profiles of three types of the transmission line.



Fig. 3. A cluster of curves between impedance of the probe and volumetric soil water content θ_v with f = 50, 100, 200 MHz, respectively, based on the achieved model.

SOME ELECTRIC PROPERTIES OF MULTI-PIN PROBE FROM THE MODEL

Either Eqs (12) or (13) show that the impedance of the soil probe changes not only depending on the dielectric constant ε , but also on the *L* as well as the *f* when the geometric parameters *D* and *d* are invariable. In order to describe the relationship between volumetric soil water content θ_{ν} and the impedance of the soil probe, the Topp equation should be used:

$$\varepsilon = 3.03 + 9.3\theta_v + 146\theta_v^2 - 76.7\theta_v^3. \tag{14}$$

Some computation results are presented in Fig. 3 under L= 10 cm, and in Fig. 4 under f= 100 MHz. From the computation results a series of important electric properties of the soil probe can be drawn.

1. Either Eqs (12) or (13) are suitable for all types of soil probes in accordance with $1 \le \xi \le 2$. In case of $\xi = 1$, the soil probe is just as for a capacitance sensor because the shape of the capacitance sensor is similar to a segment of typical coaxial line. In case of $\xi = 2$, a soil probe with 2-pin is like a segment of parallel line. When ξ is between 1 and 2, both formulas are suitable for any type of multi-pin probes. Thus, all electric properties generated by each universal formula have a broad sense.

2. In terms of multi-pin probe, ξ may be determined by calibration using each equation because of $\varepsilon \approx 1$ in air.

3. There exists a series of discontinued dots with periodical $n\pi$, n = 1, 2, 3... In order to avoid this unwanted case, a limitation should be added as below:

$$\frac{2\pi f l \sqrt{\varepsilon_{\max}}}{c} \prec \pi \Leftrightarrow f l \sqrt{\varepsilon_{\max}} \prec \frac{c}{2}.$$
 (15)

For example, from the Topp equation, it is known $\varepsilon \Big|_{\theta_V = 50\%} \approx 34$, choosing f = 100 MHz and $c = 3 \cdot 10^8$ m s⁻¹

in Eq. (15), L, i.e., the length of the pin, should be less than 0.26 m. If f is chosen at 200 MHz, L should be limited within 0.13 m.



Fig. 4. A cluster of curves between impedance of the probe and volumetric soil water content θ_v with L = 5, 10, and 15 cm, respectively, based on the achieved model.

4. Under the condition of:

$$0 \prec \frac{2\pi f l \sqrt{\varepsilon}}{c} \prec \frac{\pi}{2} \Leftrightarrow 0 \prec f l \sqrt{\varepsilon} \prec \frac{c}{4}, \qquad (16)$$

the impedance of the soil probe is equivalent to a capacitance. Similarly, if the condition satisfies:

$$\frac{\pi}{2} \prec \frac{2\pi f l \sqrt{\varepsilon}}{c} \prec \pi \Leftrightarrow \frac{c}{4} \prec f l \sqrt{\varepsilon} \prec \frac{c}{2}, \qquad (17)$$

the impedance of soil probe is equivalent to an inductance.

MATERIALS AND METHODS

The objective of the experiment was to prove Eqs (12) or (13) with different soil moistures from air dry to saturation. The soil sample used was clay-loam and its textural compositions were, sand 27%, clay 36%, and silt 37%. Measurements were taken in a laboratory at room temperature, approximately 22°C. Eight different volumetric soil water contents were examined ranging from 0 to 42.4%. The schematic diagram of the circuit to measure the impedance of the soil probe is shown in Fig. 5. The probes tested had two, three, four and five-pins. The span between the inner- and the outer-pin was 2 cm and the diameter of the pin was 0.3 cm. All pins were made of copper.



Fig. 5. Schematic diagram of the circuit to measure the impedance of soil probe.

In Figure 5 V_a or V_b stands for the output signal of each wave detector connected to point a or b, respectively. Because in most cases it is not easy to determine the value of the output impedance of a high-frequency-oscillator, additional impedance Z_o was erected in the circuit so that the Z_p could be calculated by Eq. (18).

$$Z_p = \frac{Z_o}{V_a - V_b} V_b. \tag{18}$$

Thus, the value of Z_p is independent of the output impedance of the high-frequency-oscillator in terms of Eq. (18). Since the experiments aimed at checking the computation results shown both in Figs 3 and 4, the three different lengths of the pins were 5, 10 and 15 cm and the frequencies of the oscillators were chosen at 50, 100, and 200 MHz.

RESULTS

Effect of different measurement frequencies

Figure 6 shows the measurement results concerned with the relationship between the impedance of a soil probe with 3-pin versus θ_v referring to L = 10 cm at f = 50, 100, 200MHz, respectively. From Fig. 6 it is clear that, in general, the varying procession of the impedance of the tested probe is quite similar to the theoretical computation results given in Fig. 3. When θ_v is over 25%, there is little difference among the three tested frequencies. In particular, seven measurement data referring to f = 200 MHz became negative values when $\theta_{\rm v}$ was over 33% and four measurement data referring to f = 100 MHz changed their sign at $\theta_v = 42.4\%$; these facts revealed that the impedance of the tested probe was changed from a capacitive-reactance into an inductive-reactance as θ_v increased. Instead, all measurement data referring to f =50 MHz had kept their signs until the soil samples became saturated.



Fig. 6. The measured relationship between impedance of the probe and θ_v referring to f = 50, 100, and 200 MHz.

Effect of different lengths of the pins

Figure 7 gives the measurement results to display the effect of different lengths of the pins. The measurement frequency was chosen at 100 MHz. It is noted that six measurement data associated with L=15 cm became negative values when θ_v was over 33% and two measurement data associated with L=10 cm changed their signs at $\theta_v = 42.4\%$. These measurement results confirmed again that the reactance property of the soil probe could be changed as measurement data associated with L = 5 cm kept their signs with different soil moistures from air dry to saturation.

Effect of the number of pins

As analysed above, either Eqs (12) or (13) can be viewed as a universal formula because they are suitable for all types of multi-pin probes. Experiment 3 attempted to



Fig. 7. The measured relationship between impedance of the probes and θ_v referring to L = 5,10, and 15 cm.

investigate whether Eqs (12) or (13) fits those probes with different pins. In fact, if a soil probe has *n*-pin (*n*>4), it will be inappropriate to use for most field soils because the resistant force will increase considerably during the probe's insertion into the soil. The soil sample could certainly be devastated in the sampling procession. Therefore, the pins of the probes tested in this experiment were two-, three-, four-, five-pin, respectively. Figure 8 illustrates a comparison of the impedance of each probe versus θ_v ranging from 0 to 42.4%. Actually there was little difference under the condition of *n*>3 and once θ_v is over 25%, all measured data referring to distinct θ_v converged together. By calibration in air, a series of values of ξ , which was introduced in Eq. (10), were obtained and Fig. 9 describes the relationship between ξ and the number of pins.

INSTRUMENT DESIGN

Based on the principle of this model, a prototype was designed (Fig. 10). The volume of the prototype was $9.5 \ 15 \ 5 \ \text{cm}^3$ and its power consumption was about $0.5 \ \text{W}$. The schematic diagram of the prototype circuit is given in Fig. 11. Compared with the impedance measurement circuit



Fig. 8. The measured relationship between impedance of the probes and θ_{ν} referring to the number of pins, n = 2, 3, 4, 5.



Fig. 9. The measured relationship between ξ and the number of pins.



Fig. 10. The prototype based on the model of the impedance of soil probe.



Fig. 11. Schematic diagram of the prototype circuit.

shown in Fig. 5, it is easy to understand that this circuit was extended from that one. In order to improve the performance of the probe, two additional components Z_N , which consisted of a linearizer, were parallel to the inner- and outerends of the pins. The probe used in this prototype had three pins with L = 6 cm. Although the measurement error of the prototype was up to $\pm 2\%$ for most agricultural soils θ_v ranging from 0 to 50%, it costs much less than TDR. Figure 12 shows that the non-linear relationship between the impedance of the probe and θ_v was notably corrected but the detective sensitivity of the probe was somewhat deteriorated after the linearizer was incorporated.



Fig. 12. Corrected non-linear relationship between the impedance of the probe and θ_{ν} by using Z_N .

CONCLUSION

The results of this study demonstrated that the impedance of a multi-pin soil probe could be calculated by a model derived from the theory of the impedance-transformation of transmission line. The trick to compute the characteristic-impedance of multi-pin probe was also discussed. By computing this model, some important electric properties of the multi-pin probe were discovered and the experiment results favorably approximated the theoretical analysis. In addition, a new measurement method based on this model to determine soil moisture was presented and a prototype was designed. The model discussed in this paper has great potential to assess water content for grains and other porous media.

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